Lab 2 Report

KittiCopter Control System

Dylan Trowsdale

EEE3094S Control Systems Engineering

TRWDYL001

Department of Electrical Engineering

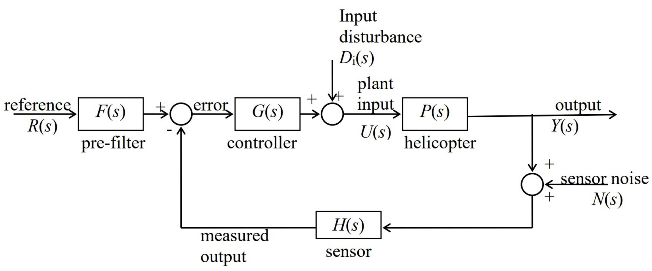
August 15, 2024

Contents

1 Introduction

The aim of this lab is to determine the specific system model of ... using ... methods. This is called System Identification. The following figure is from the Lab Instruction Sheet [UCT] and shows the Helicopter System Feedback Components in a block diagram.

The aim of this lab is to determine the specific system model of the virtual Kitticopter assigned to us using free-body force diagrams, differential equations, and block diagrams to help identify the system being modeled. These methods will allow us to determine the open and closed loop transfer function of the virtual Kitticopter. From the system model, we are required to design a proportional controller for the Kitticopter in order to meet a set of certain specifications. This lab is carried out in two steps, firstly the system identification is done by performing step responses on our system to determine the particular values of the transfer function modeled on the system. Once the transfer function is determined, in the second step, a proportional controller can then be designed for the system according to a set of design specifications. Lastly, the controller designed for the system is then tested in the lab to meet the design specifications.



2 System Modelling

model and assumptions: The virtual kitticopter is acted upon by 3 forces: rotor thrust - the input to the system, aerodynamic drag, and gravity. All the parameters are normalized to the mass of the kitticopter (unit mass). The aerodynamic drag is modeled as being directly proportional to the velocity of the kitticopter but in the opposite direction. b will stand in place for the proportionality constant. Gravity can be modeled as an input disturbance to the system. Since it is a known disturbance it can be counteracted by a constant offset input force. The Kitticopter is equipped with a sensor that perfectly measures its velocity, thus the output must be integrated to give output displacement. To allow for electronic control, the Kitticopter's position must be converted from a value in meters to a voltage by the feedback sensor. The relationship between the position and the voltage is a simple scalar that needs to be determined experimentally. The command input from the controller to the Kitticopter will also be in the form of a voltage. It is assumed that the voltage is directly proportional to the rotor thrust it instigates.

2.1 Methodology

1. model the system using a block diagram, free-body force diagram, and differential equation
2. determine the open and closed loop transfer function of the system from the differential equation
3. perform experimental step responses on the system to determine the particular values of the closed loop transfer function
4. determine the particular values of the transfer function and validate the system model using Matlab
5. design a proportional controller according to the design specifications
6. build the controller circuit and perform test procedures for the controller in the lab.

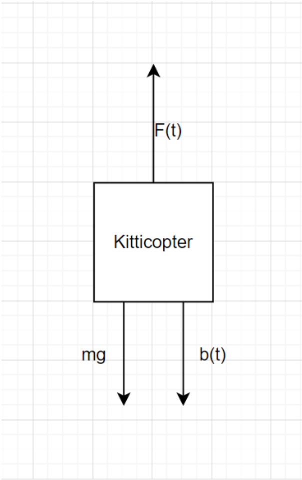


Figure 1: Free body force diagram of the Kitticopter

2.2 Modelled System and Results

2.2.1 A free-body diagram illustrating the forces acting on the Kitticopter

2.2.2 A block diagram of the whole system showing the various signals

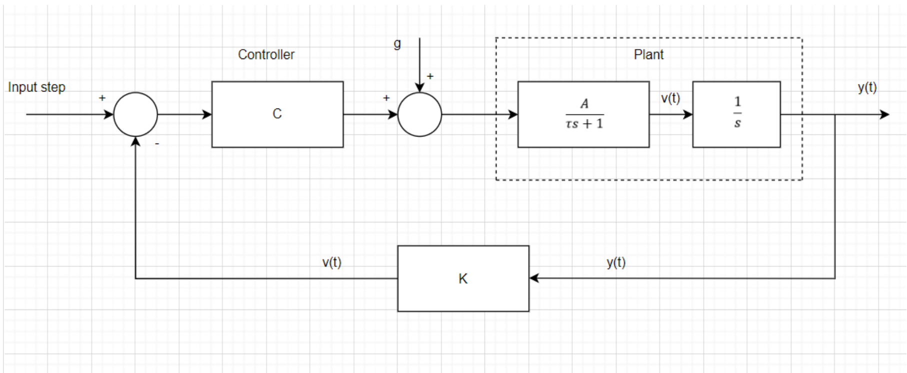


Figure 2: Block diagram of the model of the Kitticopter

Gravity is applied between the controller and the plant since it is modeled as an input disturbance. An integrator is modeled within the plant to convert the velocity to a position value at the output. The system is placed in feedback with a sensor that converts a position to a voltage.

2.2.3 A differential equation describing the motion of the Kitticopter :

Using the free body force diagram, we can use Newton's second law of motion to describe a differential equation of the system. Even though there is an acceleration in this differential equation, we are interested in the velocity. Acceleration is the first derivative of the velocity. Thus this differential equation is effectively a first-order differential equation.

$$\begin{aligned}
F &= ma \\
F_{\text{net}} &= F_T - F_B - F_G \\
m\ddot{y}(t) &= -b\dot{y}(t) + F(t) - mg \\
unitmass: & \\
\ddot{y}(t) &= -b\dot{y}(t) + F(t) - g \\
L^{-1}: s\dot{Y}(s) &= -b\dot{Y}(s) + F(s) - \frac{g}{s} \\
s\dot{Y}(s) + b\dot{Y}(s) &= F(s) - \frac{g}{s} \\
\dot{Y}(s)(s+b) &= F(s) - \frac{g}{s} \\
\dot{Y}(s) &= \frac{F(s)}{s+b} - \frac{g}{s(s+b)}
\end{aligned}$$

2.3 A transfer function derived from the differential equation

$$\frac{\dot{Y}(s)}{F(s)} = \frac{1}{s+b}$$

If we write the transfer function in standard form:

$$\frac{\dot{Y}(s)}{F(s)} = \frac{A}{\frac{1}{b}s+1} = \frac{A}{\tau s+1}$$

where A is some gain value and τ is the time constant of the transfer function.

3 System Identification

To perform System identification for our model, we first need to perform a step response. The step response of our system will allow us to determine the time constant and the steady state value for the velocity and thus the gain of the model.

To counteract the constant input disturbance of gravity applied to the system, a constant input reference voltage of 2.5V is needed. In the simulation, this is the point when the helicopter begins to hover (Ft = Fg + Fb). To perform the simulated step response, the helicopter was allowed to hover at an arbitrary height (input reference voltage = 2.5V), then an input step voltage could be applied to the system. The step voltage is then applied to the system using a voltage divider connected to a potentiometer. Due to the limits of the ADC used in the lab, the input step saturates to a voltage of 5V and the output voltage saturates to 10V. Because a potentiometer is used to apply the step to the system, it is impossible to achieve a perfect vertical input step however the best effort was taken to achieve as good of an input step as possible.

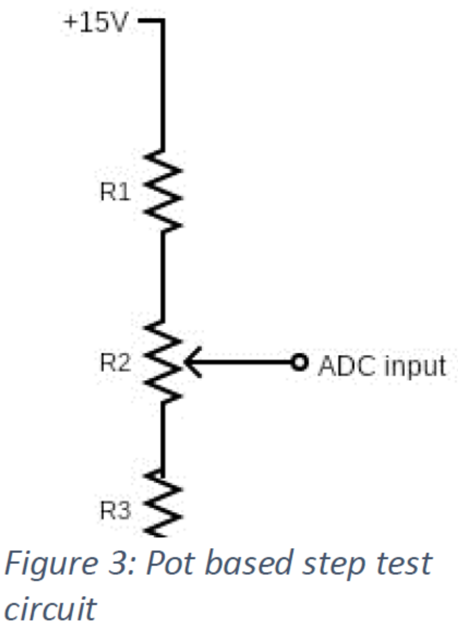


Figure 3: Pot-based step test circuit

Three step-responses were performed on the system so that an average value of the time constant and gain can be used. Using an average value takes into account for uncertainty of the step input.

3.1 Figures and Tables

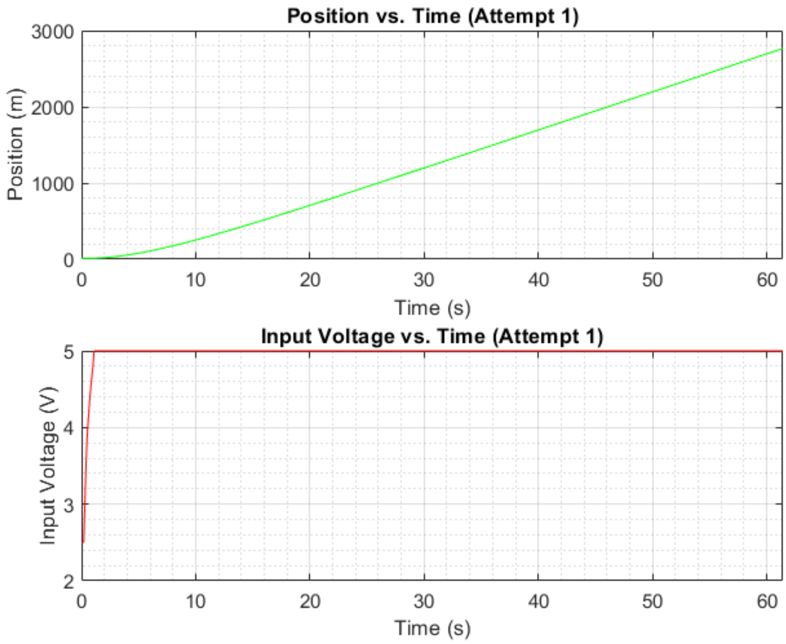


Figure 4: Step response - input step and displacement (First Attempt)

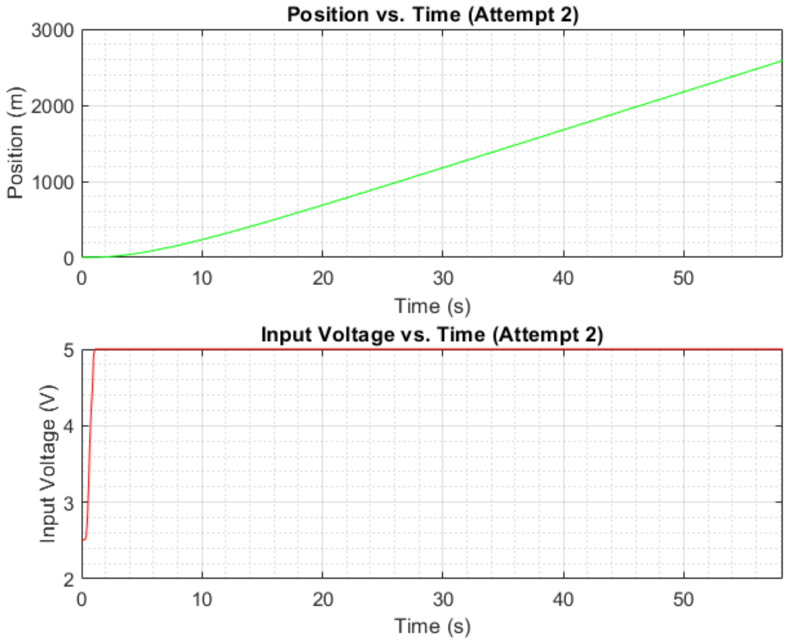


Figure 5: Step response - input step and displacement (Second Attempt)

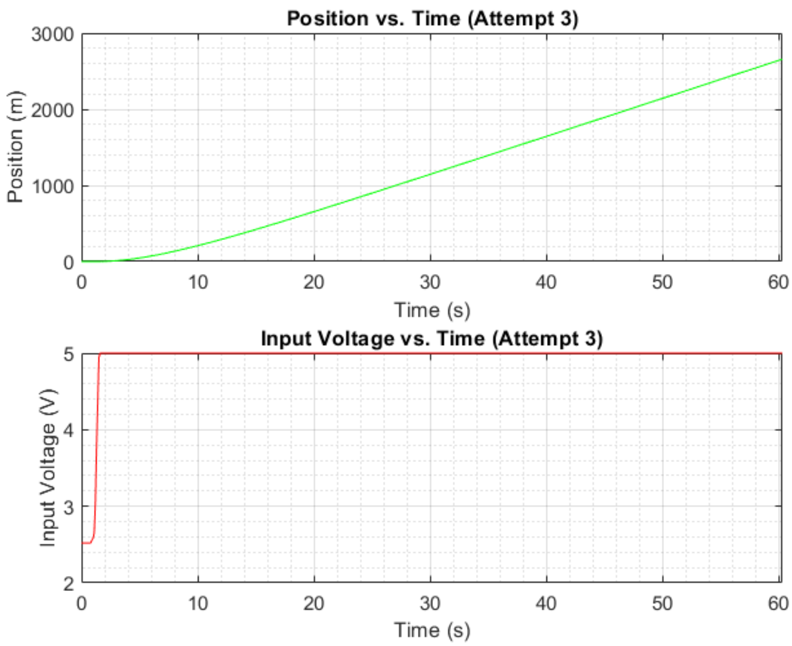


Figure 6: Step response - input step and displacement (Third Attempt)

3.2 Step Response Test

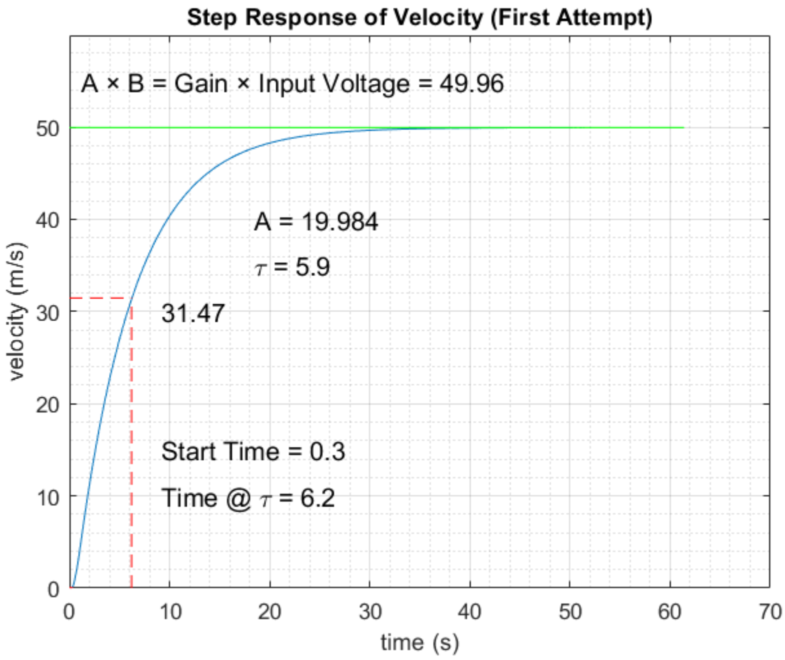


Figure 7: Step response - velocity (First Attempt)

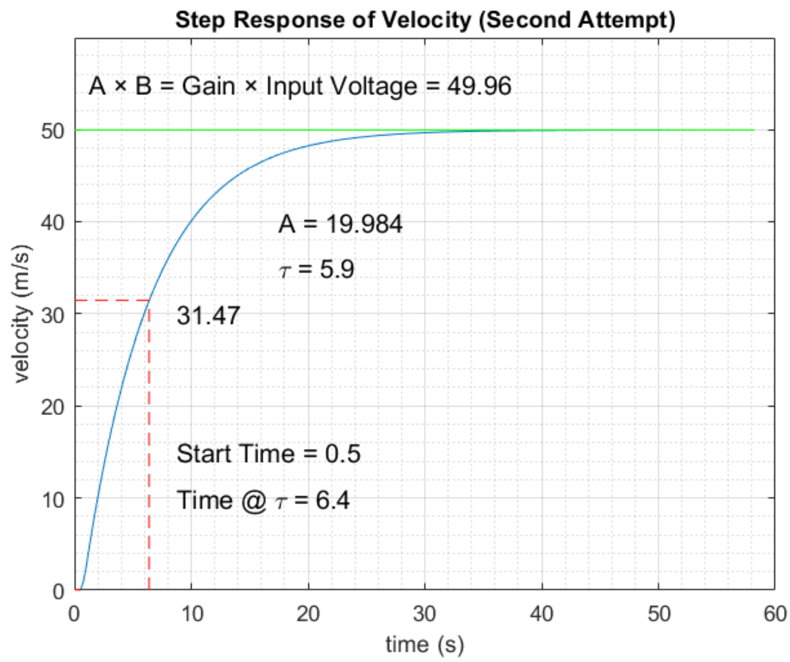


Figure 8: Step response - velocity (Second Attempt)

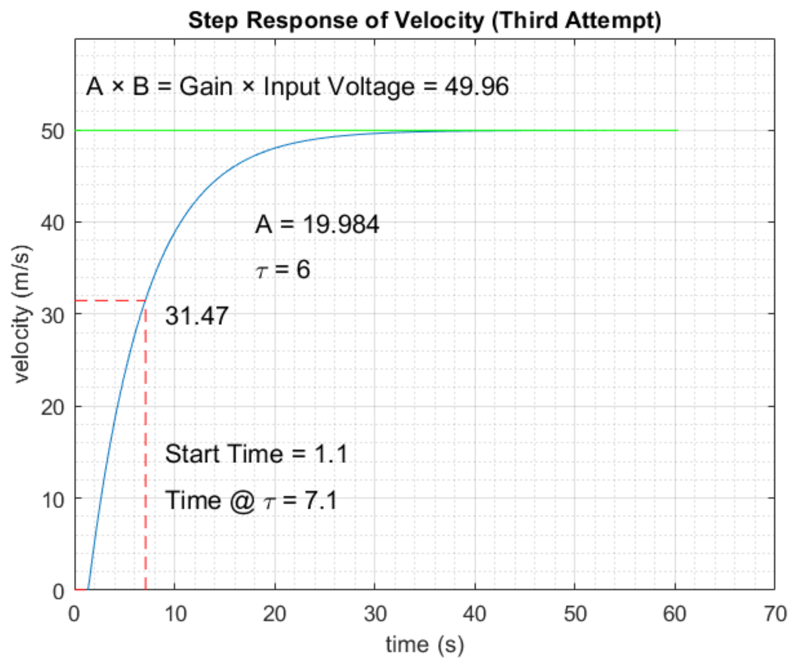


Figure 9: Step response - velocity (Third Attempt)

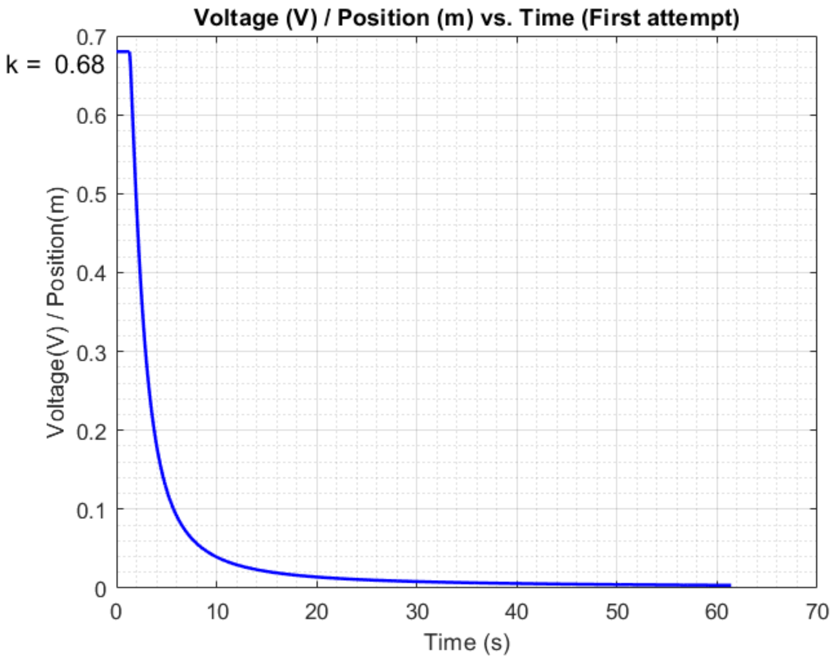


Figure 10: Determining proportionality constant m-V (First Attempt)

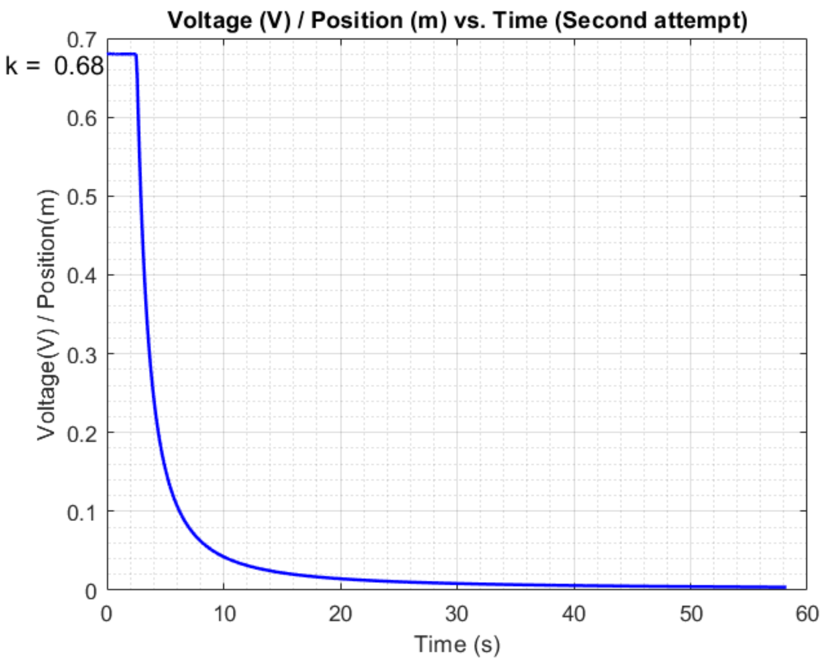


Figure 11: Determining proportionality constant m-V (Second Attempt)

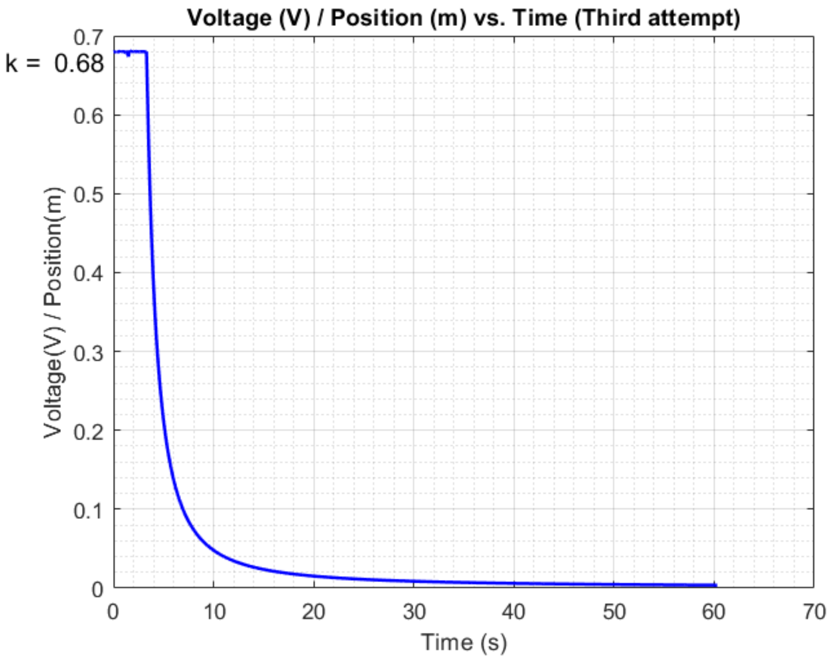


Figure 12: Determining proportionality constant m-V (Third Attempt)

Calculation of the System Identification Values:

Using Matlab the time constant and the gain of the transfer function was determined. The time constant was determined by finding the time at 63% of the steady-state value of each of the step responses. The gain A was determined by dividing the steady-state value of the step response by the magnitude of the input step used, which for the simulation was $5V - 2.5V = 2.5V$.

As seen in the three-step response trials, the time constant τ was calculated at 5.9s, 5.9s, and 6.0s respectively. Because the input step of trial 3 was not as steep of a step as the step in trials 1 and 2, it is likely to be the reason for the slightly slower time constant. Thus $\tau = 5.9s$ will be used for the Transfer function. From the derivation of the Transfer function: $\tau = \frac{1}{b}$, Therefore $b = 1/5.9 = 0.1695$

The gain value A calculated for each step response was the same: $\frac{Steady-State}{2.5} = \frac{49.96}{2.5} = 19.984V$

The proportionality constant k between the output position (m) and the input voltage (V) was calculated in Matlab by determining the transfer function between the output position and the input voltage: $V = ky; k = \frac{V}{y}$. From all three attempts shown in the figures above: $k = 0.68\frac{V}{m}$.
The closed loop Transfer function of the system is thus: $\frac{19.984}{5.9s+1}$

3.3 Model Validation

In order to validate the system, the step response of the system's modeled transfer function is plotted in Matlab and compared to the actual step responses of the system measured in the lab.

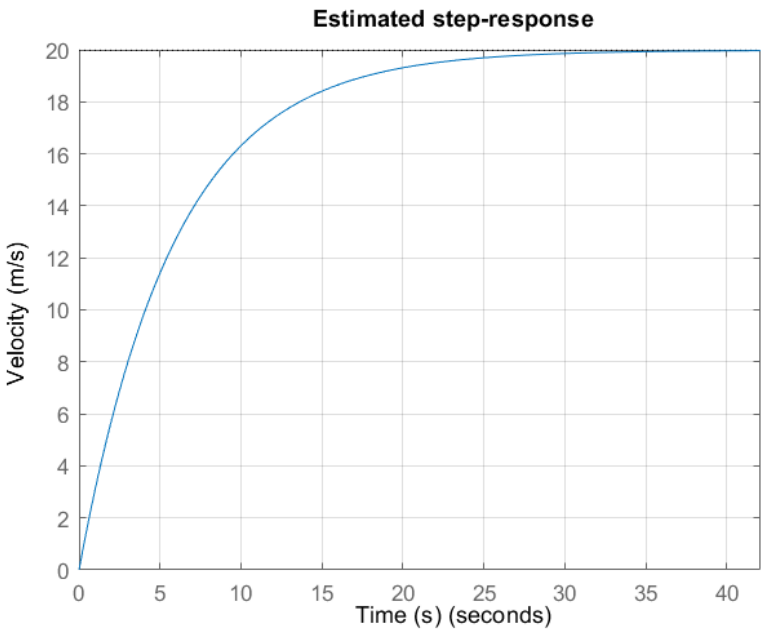


Figure 13: Estimated step response of the modeled transfer function in Matlab.

As seen from the data and the plots above, it is clear that the modeled transfer function is accurate and replicates the actual system precisely. The validated closed-loop transfer function: $\frac{19.984}{5.9s+1}$ will thus be used for the remainder of this lab.

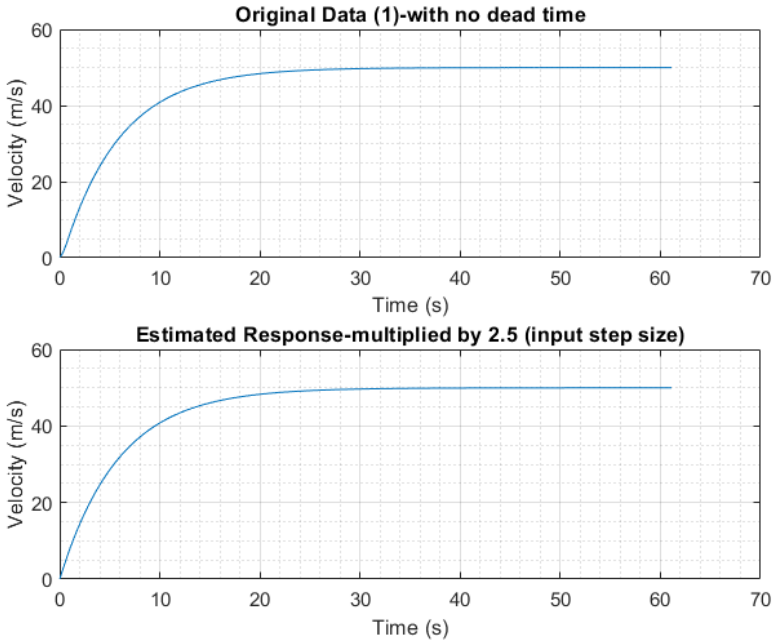


Figure 14: Comparison of the actual step response of the system measured in the lab to the estimated step response in Matlab.

4 Controller Design

4.1 Specifications

- Tracking of position inputs with greater than 90% accuracy (i.e. the tracking error and effects of disturbances must be 80% accuracy.
- Settling time improvement of at least 20% compared to the open-loop system.
- Overshoot of less than 5%.
- Robustness to uncertainty of up to 10% in the aerodynamic constant of the system.
- Robustness to a tolerance of 10% in the components used to assemble the controller.

4.2 Design

The plant transfer function is $(\frac{19.984}{5.9s+1})(\frac{1}{s})$

Let $G(s)$ represent the transfer function of the proportional controller to be designed. Since we are using a proportional controller, $G(s) = C$ where C is some gain constant. Thus, the system's open-loop transfer function will be $(G(s))(\frac{19.984}{s(5.9s+1)}) = C(\frac{19.984}{s(5.9s+1)})$

4.3 Root Locus

The root locus is a graphical representation of the s-plane that allows us to examine how the poles and zeros of the closed loop transfer function will change as one of the parameters of the open loop transfer function is varied. To design our proportional controller to meet the design specifications for our specific system we will vary the gain C of the open loop transfer function. The following Root locus of the open-loop transfer function was plotted in Matlab: From the root locus, we can see that there are two open-loop poles in the

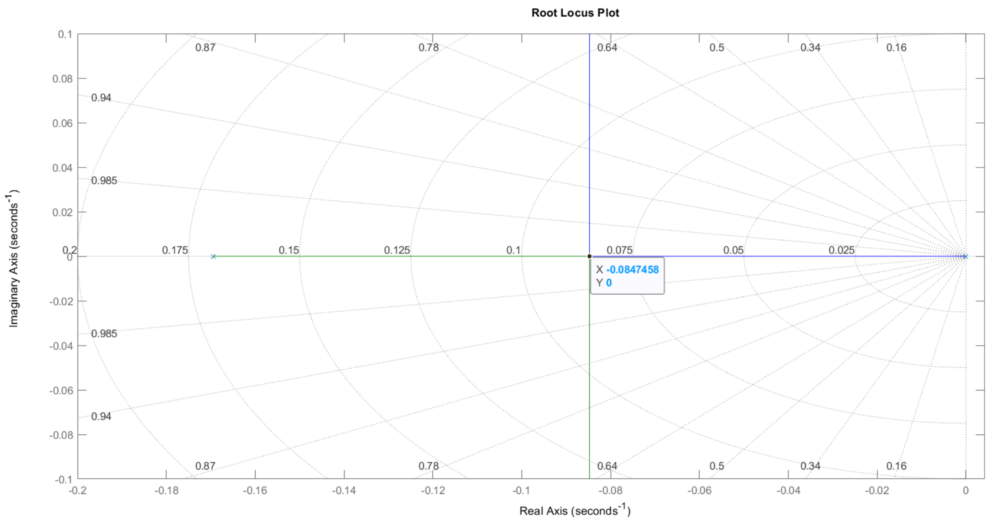


Figure 15: Kitticopter root locus

left half plane, one at the origin and one at -0.1695. Since there are no open-loop poles or zeros in the right half plane, the system cannot become unstable for any value of gain C.

Since we have added an integrator to our system so that we can get the output displacement from the velocity measured by our sensor, we have introduced a pole at the origin. Since the pole is at the origin it becomes more dominant and it causes the settling time to be extremely high. From the Root Locus plotted in Matlab above, we can see that the highest gain that the system can have before it becomes oscillatory will be at the centroid between the two open loop poles. This is the point where the closed-loop system will be critically damped i.e. the highest gain before the system becomes oscillatory and the fastest response possible for the closed-loop system. At this critically damped point,

$\tau = |\frac{1}{-0.0847458}| = 11.80s$ To determine the gain at this point we can do a calculation:

$$
\begin{aligned}
|G(s)P(s)| &= 1 \\
|C(P(-0.0847458))| &= 1 \\
C| - 471.6224| &= 1 \\
C &= \frac{1}{471.6224} \\
C &= 0.0021203
\end{aligned}
$$

4.4 Tests performed on the Controller

4.5 Validation

This gain value of the proportional controller was first tested in Simulink to determine if the system at this gain value would meet the specifications. As seen from the

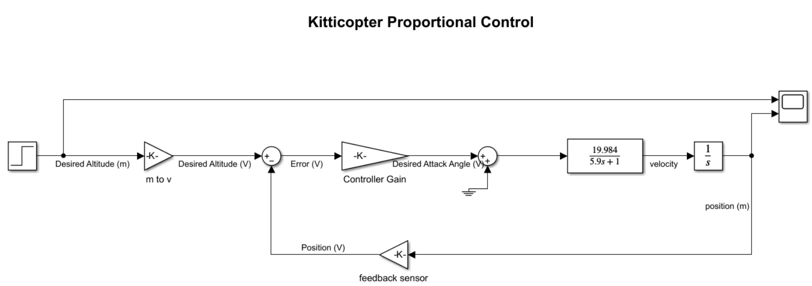


Figure 16: Simulink model of the Kitticopter system

simulation of the system with the proportional controller the system now tracks an input step perfectly with 100% accuracy and with no overshoot at all. As previously explained, since the integrator added into the system adds an open loop pole at the origin, this pole will dominate the response of the open loop system. Therefore, from the root locus shown above, any gain above 0 will cause the system to become faster than the open loop system

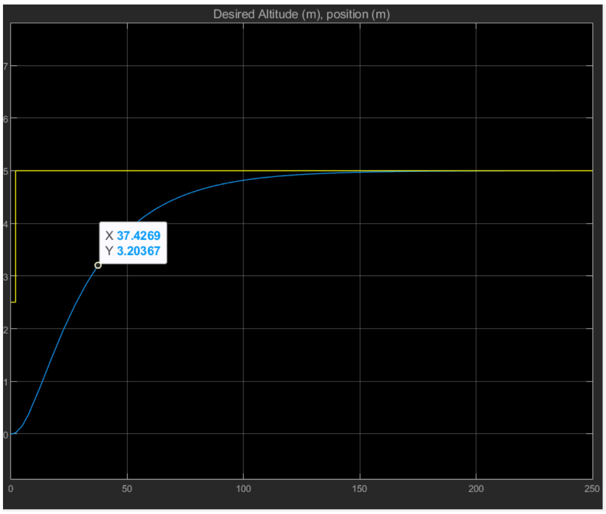


Figure 17: Simulink simulation scope

to a maximum theoretical time constant of $\tau = 11.80s$. From the Simulink simulation, the system did not reach this theoretical value which could be due to the scaling factors introduced by the conversions between voltage and meters from the output (m) to the input (V) when connected in feedback. However, the response time is still much faster than what it would be in an open-loop with a pole at the origin.

4.6 Circuit design

Following overall feedback op-amp circuit design was chosen for the controller. The overall circuit consists of a subtracting junction (Differential amplifier) that sets a reference voltage to be tracked, a gain amplifier (inverting amplifier) used to set the controller gain C, and lastly, an inverting summing amplifier used to provide the offset to counteract the constant input disturbance of gravity. The inverting summing junction must be used to invert the negative voltage gain of the inverting amplifier back to a positive voltage gain at the output. All the resistors used in the amplifier circuits for the correct amplifier gain were selected to be 560K ohm resistors. Each resistor was measured in the White lab and the chosen resistors had resistances that were at most off the desired value of 560K by 20 ohms. This leads to a maximum error of $\frac{20}{560000} \times 100 = 0.0036\%$. This is an extremely small tolerance and can allow for the amplifier gain to be extremely precise and consistent.

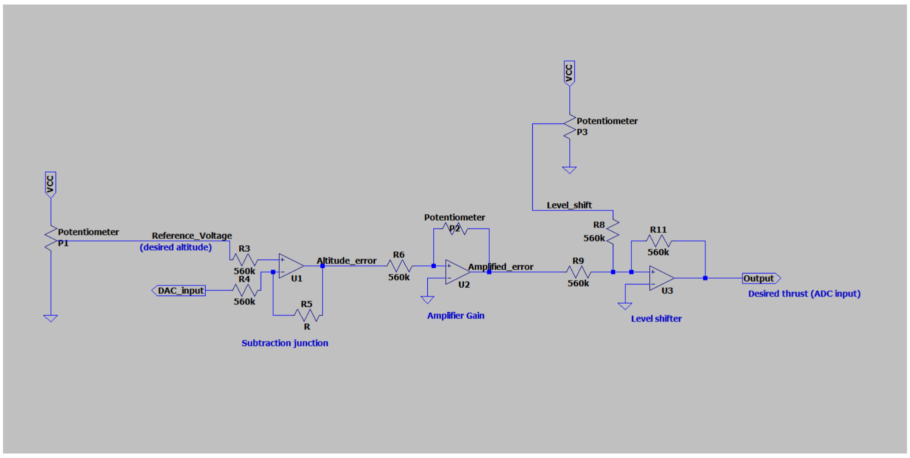


Figure 18: Controller circuit

4.7 Calculations

To achieve the desired gain of $C = 0.0021203$ we are required to set the amplifier gain of Inverting amplifier to this value. For an inverting amplifier:

$$Gain = \frac{Rf}{Ri}$$
$$0.0021203 = \frac{Rf}{560000}$$
$$Rf = 0.0021203 \times 560000$$
$$Rf = 1321.81 ohms$$

Therefore we can use a 10K pot for resistor Rf because we will likely need a higher gain than this in practice and a 10K pot should have enough headroom for gain adjustments in practice.

4.8 Lab procedure

The circuit above was built on a breadboard first and connected to the simulation program in the controls lab to validate that the circuit works in practice and that the helicopter actually tracks the set point given. Once the breadboard circuit was connected to the simulation platform in the controls lab, it was verified that the system tracks a given set point, however, gain adjustments would need to be made to meet the design requirements. The circuit was then assembled and soldered on a piece of Veroboard after which the Controller testing and evaluation could be done.

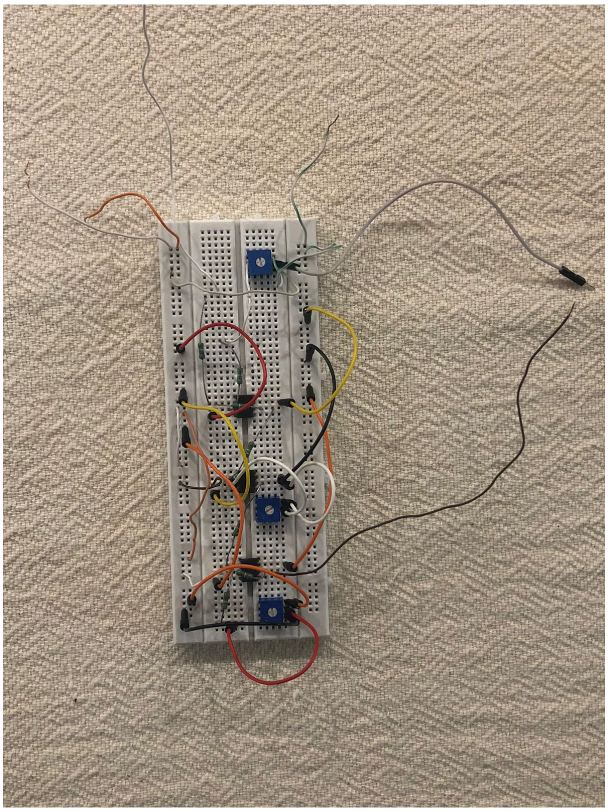


Figure 19: Controller circuit on the breadboard

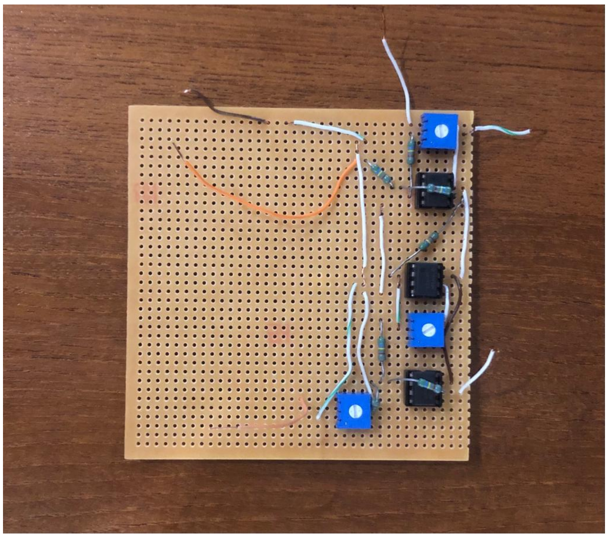


Figure 20: Controller circuit on the veroboard

5 Controller Testing

5.1 Performance Evaluation

Once the circuit was assembled and soldered on the veroboard, the controller was tested in the simulation program in the control lab. Initially, the gain set by the potentiometer on the breadboard was set to a low value estimated to be close to the theoretically required resistance of 1321.81 ohms. The input offset voltage was carefully set to 2.5V by turning the third pot to offset the constant offset force of gravity and the set-point altitude was carefully set to 5m by turning the first pot.

The controller was then tested at this gain value and it was able to track the set-point within the specifications, however, the settling time of the system was quite slow. The gain was then increased to the maximum gain allowed by the pot by setting its resistance to 10k ohms but the system began to overshoot the set-point that it was attempting to track and was not within specifications. Thus, the resistance of the pot was decreased slightly to lower the gain slightly from this point and the system was tested at this new gain value. The system now tracked the set-point very precisely and the settling time improved a lot from the first attempt. The results of the controller tests were as follows: From the

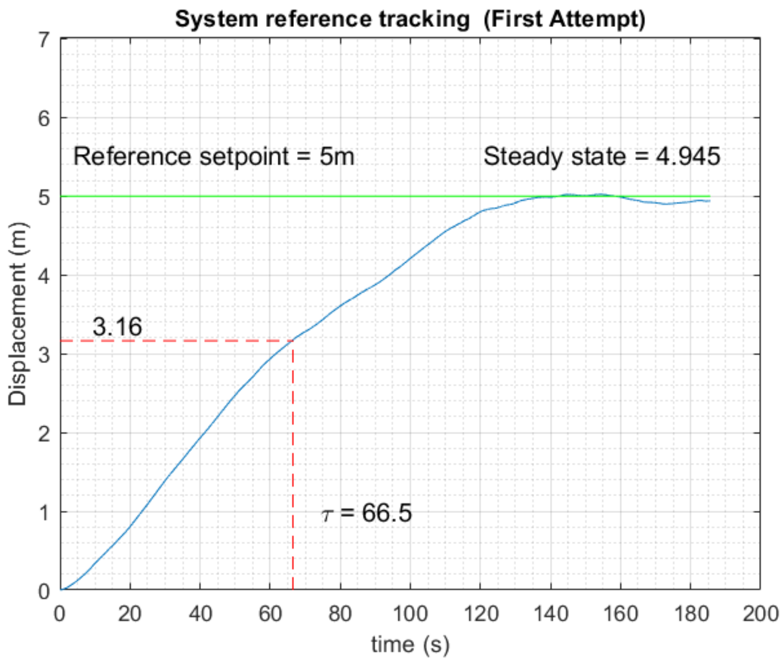


Figure 21: Controller test 1

first test, we can see that with the controller, the system tracked the 5m set point very closely. The system did not appear to behave in an oscillatory way, thus the value above 5m was assumed to be due to a tracking error. The steady-state value of the system varied between 4.945 and 5.022 V. The maximum tracking error was $\frac{4.945-5}{5} \times 100 = -1.1\%$. At this gain the system meets all the design specifications, however, an improvement of the settling time was wanted as $\tau = 66.5s$ was considered to be quite slow. The second test was an attempt to improve the settling time by increasing the gain. The second attempt

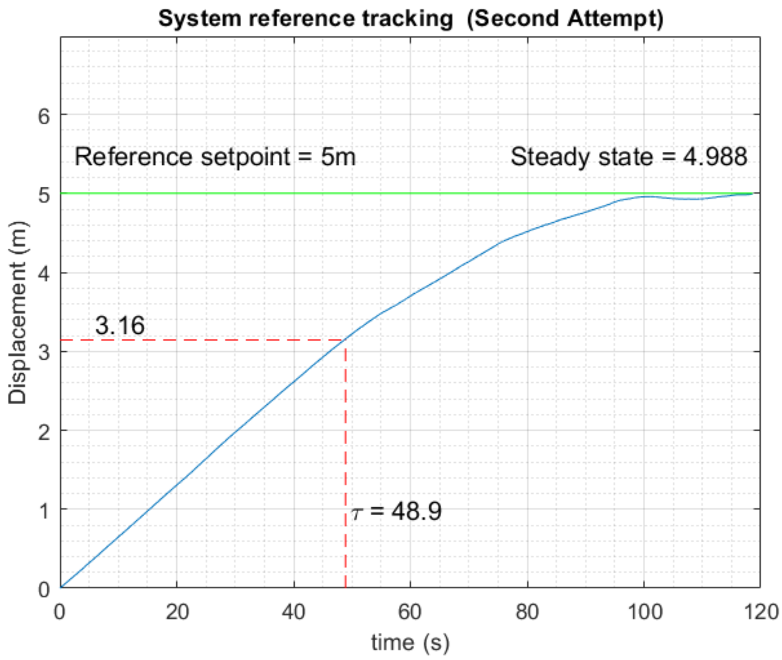


Figure 22: Controller test 2

shows that the system with the controller was able to track the reference set-point very precisely. The tracking error was $\frac{4.988-5}{5} \times 100 = -0.24\%$ which is well within the 90% tracking accuracy specification. The system did not overshoot the set point at all as well so it was also within the 5% overshoot specification. From previous explanations, the open loop system has a pole at the origin which would dominate the response, thus, the open loop system would have an extremely high almost infinite settling time. Therefore a time constant $\tau = 48.9s$ meets the 20% settling time improvement specification. As very precise and accurate resistor values for the controller circuit were chosen, the effects of component tolerances will have a negligible effect on the controller's performance and thus the system meets the specification of robustness to 10% of component values. As we have seen from both the controller tests time constant values of $\tau 1 = 66.5s, \tau 2 = 48.9s$

were achieved. The percentage change of time constant $= \frac{66.5-48.9}{48.9} \times 100 = 35.99\%$. Since the controller met all the required design specifications in both tests, the controller also meets the Robustness to the uncertainty of up to 10% in the aerodynamic constant of the system since $\tau = \frac{1}{b}$.

6 Discussion and Conclusion

From the evaluation of the controller tests, it is evident that the controller designed for the Kitticopter system works as expected and was able to meet all of the design specifications set out in this lab. The controller allowed the Kitticopter to track a reference point set by the user to an accuracy above 99% with no overshoot as well as being able to meet the settling time specifications and robustness specifications. To ensure that the robustness specifications were met, components were carefully selected. Resistor values were carefully measured using the multimeters in the white lab and the components chosen had values well under 1% different from each other. Since a pot was chosen to precisely control the gain value of the proportional controller, it is easy to make small adjustments to the gain as needed. The controller tests show that the controller still met tracking and overshoot specifications even when the settling times were vastly different. This shows that the controller is Robust to the uncertainty of much more than 10% in the aerodynamic constant of the system.

In conclusion, this report has dealt with the system identification and modeling of a virtual Kiiticopter system and secondly the design and implementation of a proportional controller aimed to allow the Kitticopter to track a user-defined set-point within certain specifications. This lab was conducted in two main steps: system identification and modeling, and secondly, the controller design and implementation.

In the first step, system identification of a model of the virtual Kitticopter system was done to determine the specific Transfer Function of the system. The system identification was done using free-body force diagrams, differential equations, and block diagrams. Step responses of the system were conducted in the lab on the virtual Kitticopter. An evaluation of the step responses allowed the determination of the particular values of the transfer function.

The second step involved designing and constructing a proportional controller for the virtual Kitticopter system to meet a certain set of specifications. Using the Root locus of the open loop transfer function and the Simulation capabilities of Simulink, a proportional controller circuit was designed for the Kitticopter. The controller circuit was then carefully constructed on a breadboard and validated in the lab. Finally, the circuit was constructed and soldered on veroboard and finally tested again in the lab.

The discussion of the lab results showed that the proportional controller designed and constructed was able to meet all of the specifications required.

6.1 Recommendations for Future Iterations

The controller design was able to meet all the specifications required, however, there were some difficulties encountered in this lab to achieve these results. Potentiometers offer an easy way to adjust the gain value of the proportional controller as necessary, however, in our system, pots were also used to control the set point and the input offset voltage. It was often very difficult to control the pot very finely to attain small exact voltages for the input offset voltage. A recommendation would be to use a more reliable method to supply the input offset for example a voltage regulator. The use of a simple proportional controller in this lab made it difficult to achieve all the specifications set out. Using a PID controller would make this process much easier and would allow for much better performance.

7 Appendix and Notes

This lab has given a lot of insight and knowledge into control engineering and has shown the limitations of proportional control.

References

[Tut] Electronics Tutorials. *The Differential Amplifier*. URL: `https://www.electronics-tutorials.ws/opamp/opamp_5.html`.

[UCT] UCT. *EEE3094S KittiCopter Control Laboratory*.

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