# Lab 3 Report

*KittiCopter Control System*

Dylan Ttowsdale

EEE3094S Control Systems Engineering

TRWDYL001

Department of Electrical Engineering

August 16, 2024

# Contents

# 1 Introduction

The aim of the previous Helicopter lab was to determine the specific system model and the open and closed loop Transfer functions of the virtual Kitticopter assigned to us. The system model was used to design a proportional controller for the Kitticopter to meet a set of certain specifications. For Lab 3, the aim is to take the lessons learned from Lab 2, especially regarding the limitations of a proportional controller, and now design a lead-compensator controller to improve upon the specifications of the system met in Lab 2.

# 2 Technical Specifications

- Tracking of position inputs with greater than 90% accuracy (i.e. the tracking error and effects of disturbances must be less than 10%)
- Settling time improvement of at least 20% compared to the performance of the uncompensated closed-loop system.
- Overshoot of less than 5%. % Overshoot = $100exp^{(\frac{\zeta\pi}{\sqrt{1-\zeta^2}})}$. Rearranging this equation, the damping coefficient,

$$\zeta \geq \sqrt{(\frac{1}{1+\frac{\pi^2}{ln^2(\frac{5}{100})}})}$$
$$\zeta \geq 0.6901$$

- Robustness to uncertainty of up to 10% in the aerodynamic constant of the system.
- Robustness to a tolerance of 10% in the components used to assemble the controller.

# 3 System Modelling

From Lab 2, using free-body force diagrams, differential equations, and block diagrams, the system was modeled and the transfer functions were determined. The system identification was completed, by performing step tests on the virtual Kitticopter system to determine the particular values of the transfer function modeled on the system.

The following System modeling was conducted on the system:

## 3.1 A free-body force diagram of the system

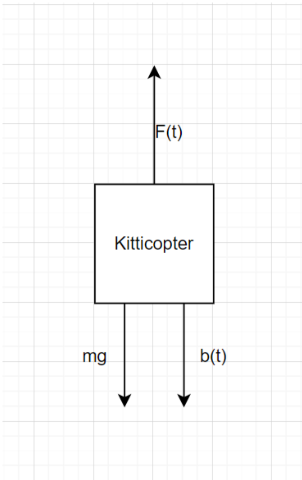


Figure 1: Free body force diagram of the Kitticopter

## 3.2 Transfer function

The Transfer function of the system:

$$\frac{\dot{Y}(s)}{F(s)} = \frac{A}{\frac{1}{b}s+1} = \frac{A}{\tau s+1} = \frac{19.984}{5.9s+1}$$

where A is some gain value and $\tau$ is the time constant of the transfer function. Since the Kitticopter is equipped with a sensor that perfectly measures its velocity, the output must be integrated to give output displacement.

Therefore, the plant Transfer function G(s) = $(\frac{19.984}{5.9s+1})(\frac{1}{s})$

Finally, to allow for electronic control, the Kitticopter's position must be converted from a value in meters to a voltage by the feedback sensor. The proportionality constant k between the output position (m) and the input voltage (V) was calculated in Matlab by determining the transfer function between the output position and the input voltage: $V = ky; k = \frac{V}{y}, k = 0.68\frac{V}{m}$. Therefore H(s) = 0.68

The modeled system is of course probably not completely accurate but it was validated in Matlab by comparing the measured step responses in the lab to the simulated step responses of the system.

## 3.3 A block diagram of the system

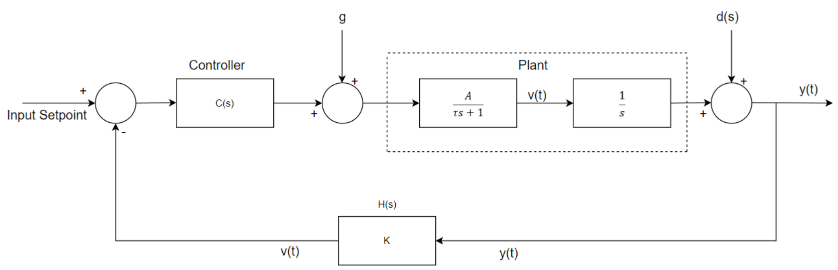


Figure 2: Enter Caption

where v(t) is the velocity of the Kitticopter, y(t) is the position of the Kitticopter, g is gravity input disturbance and d(s) are output disturbances.

## 3.4 Methodology

1. The previous system identification values from the previous proportional controller lab were used to design a lead compensator controller.
2. The lead-compensator controller is verified and simulated in Matlab.
3. The circuit for the lead-compensator controller is designed for the system.
4. The lead-compensator controller is built on a breadboard and tested in the lab.
5. The final revised lead-compensator controller is soldered on a piece of veroboard and then final controller tests are performed in the lab
6. Evaluations, conclusions, and recommendations are made on the performance of the controller and whether the specifications have been met.

# 4 Controller Design

For this lab, the aim is to improve upon the specifications achieved in the previous lab. Previously, a simple Proportional controller was used which has a transfer function of C(s) = $K$. This controller is very simple and simply adjusts the gain.

The chosen controller was a Lead-Compensator controller which has a transfer function of C(s) = $K \times (\frac{s+a}{s+b})$. where s = -a and s = -b are the desired zero and pole locations. A Lead-Compensator controller is a decent balance between simplicity and flexibility to improve performance specifications for the system.

Another type of controller includes a PI-Lead-Compensator which has a transfer function of C(s) = $K \times (\frac{s+a}{s} \times \frac{s+b}{s+c})$. This controller is also highly flexible like a regular Lead-Compensator but has some benefits as it completely rejects step input disturbances, however, its circuitry is very complex. Therefore, a regular Lead-Compensator was chosen.

The Lead-Compensator will help remedy the system's transient response as it allows for the pole of the controller to be placed to the left of the system's poles thus shifting the centroid of the root locus to the left which makes the system faster and at the same time it improves the damping ratio of the system Therefore potentially also reducing overshoot.

## 4.1 Controller Design

The plant transfer function is $(\frac{19.984}{5.9s+1})(\frac{1}{s})$

The system's open-loop transfer function will be $C(s) \times G(s) \times H(s) = K \times (\frac{s+a}{s+b}) \times (\frac{19.984}{5.9s+1})(\frac{1}{s}) \times 0.68$

## 4.2 Root Locus

The root locus is a graphical representation of the s-plane that allows us to examine how the poles and zeros of the closed loop transfer function will change as one of the parameters of the open loop transfer function is varied. To design the proportional controller to meet the design specifications for our specific system the gain of the open loop transfer function was varied. To design the Lead-Compensator, initially, the same process is done to decide the gain value that will achieve the tracking accuracy requirement.

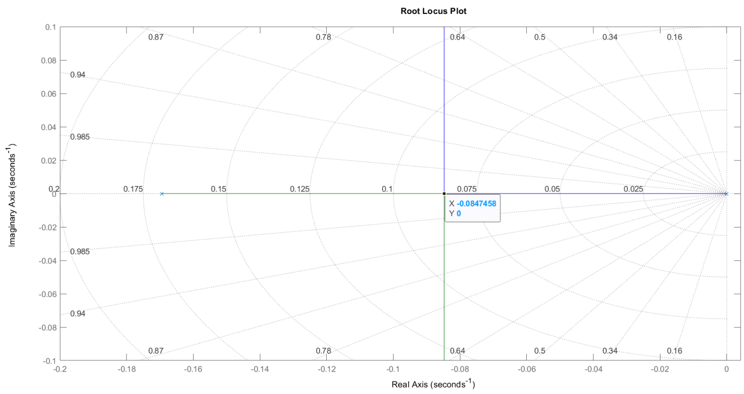


Figure 3: The following is the root locus of the uncompensated system

From the root locus, we can see that there are two open-loop poles in the left half plane, one at the origin and one at -0.1695. Since there are no open-loop poles or zeros in the right half plane, the system cannot become unstable for any value of gain C. Since we have added an integrator to our system so that we can get the output displacement from the velocity measured by our sensor, we have introduced a pole at the origin. Since the pole is at the origin it becomes more dominant and it causes the settling time to be extremely high. From the Root Locus plotted in Matlab above, we can see that the highest gain that the system can have before it becomes oscillatory will be at the centroid between the two open loop poles. This is the point where the closed-loop system will be critically damped i.e. the highest gain before the system becomes oscillatory and the fastest response possible for the closed-loop system. At this critically damped point, $\tau = |\frac{1}{-0.0847458}| = 11.80s$

In the previous lab for the proportional controller, this was the desired settling time for our system. For the Lead-Compensator we are aiming to achieve a 20% improvement over the setting time from the system with the proportional controller.

calculated values:

1. Therefore $\tau = 0.8 \times 11.80 = 9.44s$. This means that the desired system closed loop poles will be at $\sigma = \frac{1}{9.44} = 0.1059$ 2. For 5% overshoot:

$$\begin{aligned}
cos(\theta) &= \zeta \geq 0.6901 \\
\frac{0.1059}{\omega n} &\geq 0.6901 \\
\omega n &\leq \frac{0.1059}{0.6901} \\
\omega n &\leq 0.1535 rad/s \\
\omega d &\leq \sqrt{\omega n^2 - 0.1059^2} \leq 0.1111 rad/s
\end{aligned}$$

Therefore the calculated CL pole: $sd = -0.1059 \pm 0.1111j$ Therefore the calculated phase of the pole is $atan(\frac{0.1111}{0.1059}) = 46.3727$

Choosing a pole at the fastest system pole: $sd = -\frac{1}{5.9} \pm 0.1111j = -0.1695 \pm 0.1111j$

Let $\theta 2 = 90$

This is not an odd multiple of 180, therefore the compensator must add the difference. Therefore compensator must add $236.76 - 180 = 56.76$

3. Therefore, max distance of zero:

$$tan(56.76) = \frac{0.1111}{d}$$
$$d = 0.0728$$

4. Therefore, try with zero halfway. Therefore, the position of zero $= -0.1695 - \frac{0.0728}{2} = -0.2059$. Therefore,

$$\theta z = atan\frac{0.1111}{0.0364}$$
$$\theta z = 71.86$$

Therefore, the position of the compensator pole $= -0.1695 - 0.4118 = -0.5813$

5. $C(s) = K(\frac{s+0.2059}{s+0.5813})$

$$|KC(sd)G(sd)| = 1$$
$$K|41.2333| = 1$$
$$K = \frac{1}{41.2333}$$
$$K = \frac{1}{41.2333}$$
$$K = 0.02425$$

Therefore $C(s) = \frac{0.02425(s+0.2059)}{s+0.5813}$

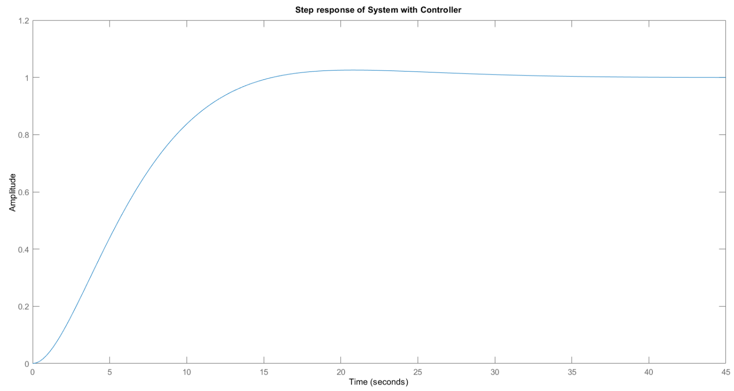


Figure 5: Step response of the system with the controller in Matlab

## 4.3 Tests performed on the controller

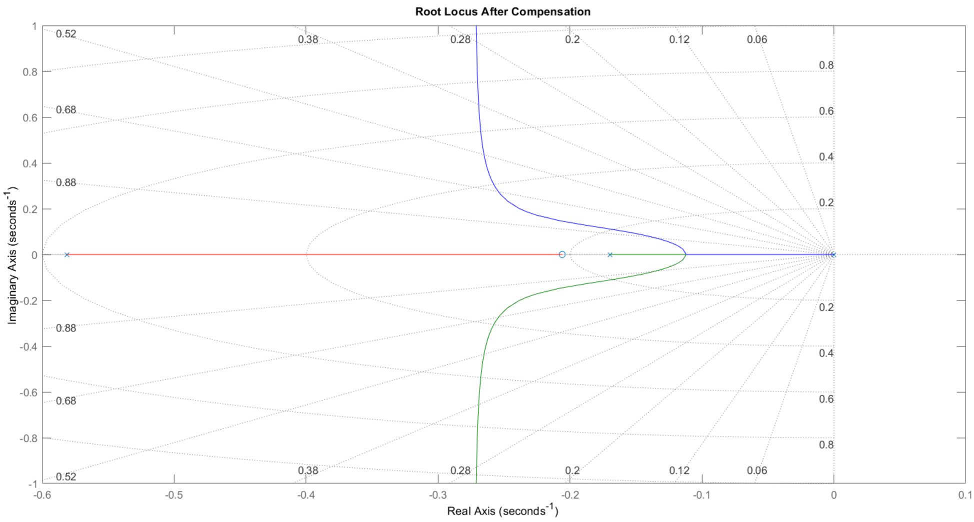


Figure 4: Root locus after compensation

The System with the lead compensator was tested and validated in simulation in Matlab. From the step response shown in the figure above it is evident that the system tracks a step within the specifications.

## 4.4 Circuit design


Figure 6: Circuit for Lead-lag compensator

Initially, the circuit was built on a breadboard using the following diagram. The capacitors were chosen to be 33uF.

$$\frac{-1}{R1C1} = -0.2059$$
$$R1 = \frac{1}{0.2059 \times C1}$$
$$R1 = 147173.53ohms$$

$$\frac{-1}{R2C2} = -0.5813$$
$$R1 = \frac{1}{0.5813 \times C2}$$
$$R2 = 52129.76ohms$$

Therefore R1 was chosen to be a 150k resistor measured with the multimeter in white lab to be 148k and R2 was chosen to be a 50k in series with 2 1k resistors. The combination of these three was measured to be 50250+960+970 = 51180 ohms. The gain of the circuit:

$$K = 0.02425 = \frac{R2}{R1} \times \frac{R4}{R3}$$
$$0.02425 = 0.34581 \times \frac{R4}{R3}$$
$$0.07013 = \frac{R4}{R3}$$

R3 was chosen to be 100k, and using the multimeter, a resistor with a value of 99.800k ohms was selected. Therefore R4 = 6998.97 ohms. Using a multimeter a resistor was chosen with a value of 6910 ohms.

Therefore the transfer function of the circuit built $C(s) = \frac{6910}{99800} \times \frac{51180}{148000} \times (\frac{148000 \times 33uf \times s+1}{51180 \times 33uf \times s+1})$ Therefore $C(s) = 0.02394 \times (\frac{4.884s+1}{1.689s+1})$

The gain was accurate to almost 3 decimal places, the zero was at s = -0.205 which is accurate to 3 decimal places and lastly, the pole is at s = -0.592 which is accurate to 1 decimal place but only 0.1 off.

This shows that in physical implementation it is difficult to attain the desired controller transfer function even when using a multimeter to carefully measure resistors to match the calculated values very precisely.

The circuit described above was then built on a breadboard and tested in the lab, however, the system did not meet any of the specifications. Many revisions were made by adjusting the gain value and adjusting the position of the pole and zero of the Lead-Compensator but the system was very oscillatory in the lab testing and had large overshoot and very slow settling time.

After much trial and error, a more robust circuit was designed with much more lea-way between the theoretical zero and pole locations and the values attained in reality.

100uF and 1000uF capacitors were chosen to be used for the circuit to allow the circuit to be built with more accessible resistor values. All of the resistors for the final circuit were measured carefully with an oscilloscope in the white lab to ensure accuracy within 1% of the stated values. This allowed the circuit to be very precise and more robust to uncertainties of component values.

The controller zero was set at $s = \frac{-1}{2.2k \times 1000uF} = -0.4545$ The controller pole was set at $s = \frac{-1}{1k \times 100uF} = -10$ The gain was set at $K = 2.7 \times \frac{1}{2.2} = 1.2273$ The zero and pole of the controller were shifted further to the left on the root locus and the gain was increased. This allows the physical implementation of the controller to be more attainable and more robust.

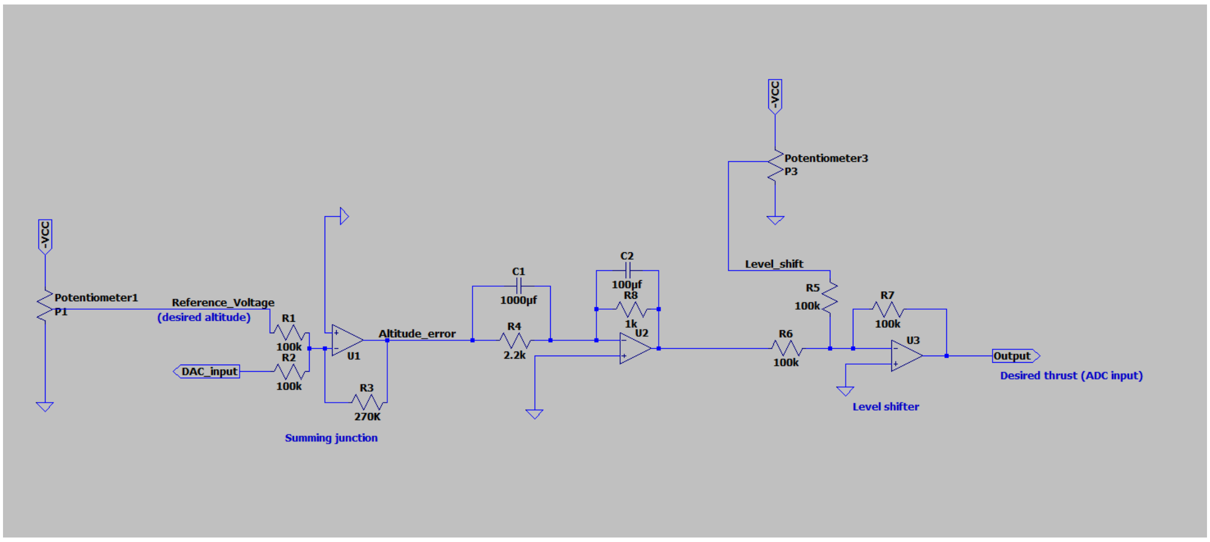


Figure 7: The final controller circuit schematic

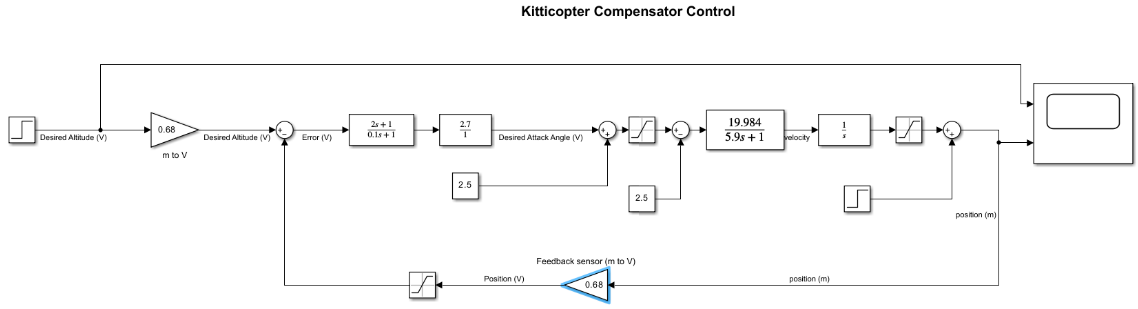


Figure 8: Simulink simulation

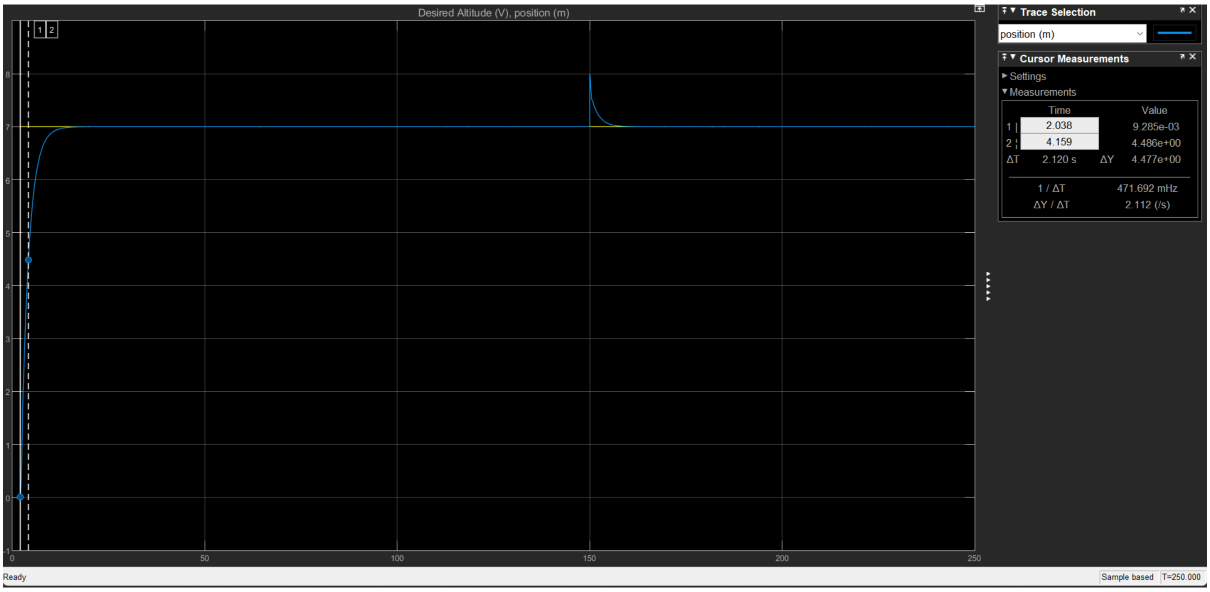


Figure 9: Simulink scope

From the Simulink scope displayed in Matlab, the system clearly was able to track the setpoint with 100% accuracy and also was able to handle output disturbances. The System has no overshoot and was able to reach the 63% time ( at 4.424 m ) in under 2.1 s. Thus, this controller is able to improve upon the specifications even further and has been proven and Validated in simulation.

The final circuit was then built on a breadboard and tested in the lab and it was able to meet the specifications. Therefore it was then decided to Solder the final Controller circuit onto a piece of veroboard and then begin the final controller tests in the lab.

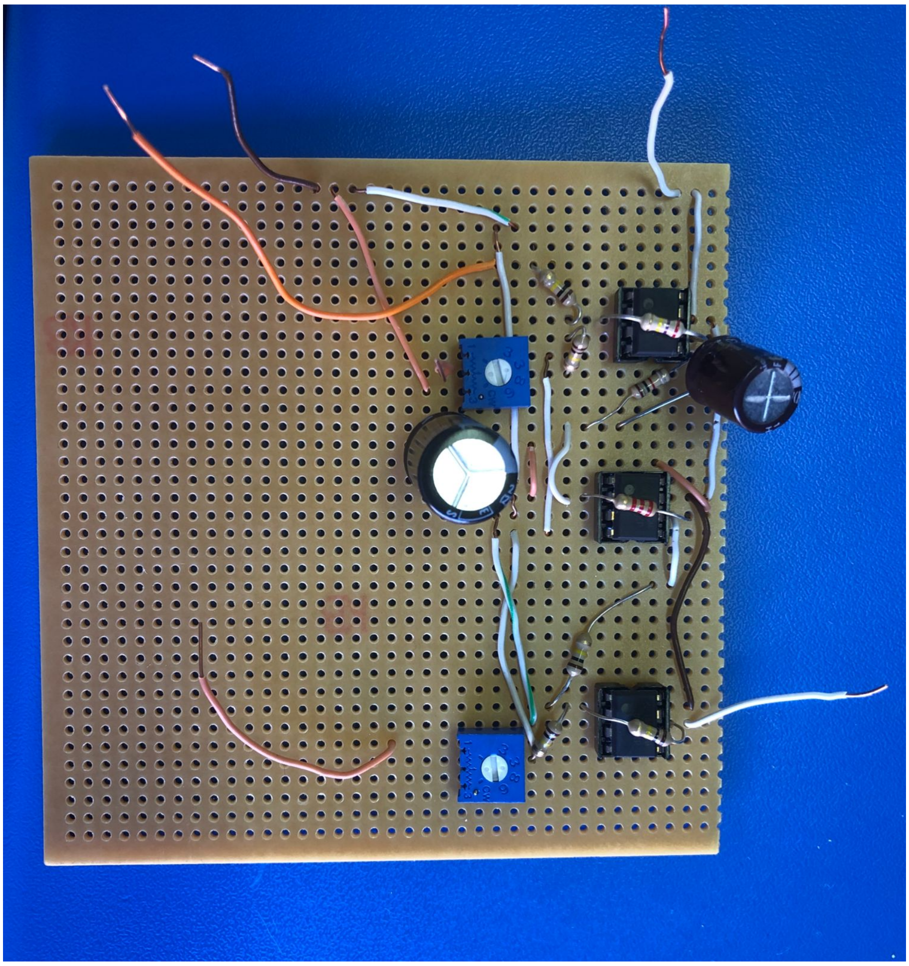

Figure 10: Veroboard circuit

# 5 Controller Testing

Once the circuit was assembled and soldered on the veroboard, the controller was tested in the simulation program in the control lab. The input offset voltage was carefully set to 2.5V by turning the third pot to offset the constant offset force of gravity whilst the setpoint was set to 0m. The set-point altitude was then carefully set to 7.5m for trials 1 and 2 and 7m for trial 3 by carefully turning the first pot. Once the setpoint and offset voltage was applied, the helicopter's engine was started and it was allowed to take off and track the setpoint.

Once the helicopter successfully tracked the given setpoint, output disturbances were applied to the helicopter in the simulation to test its robustness and ability to handle these disturbances.

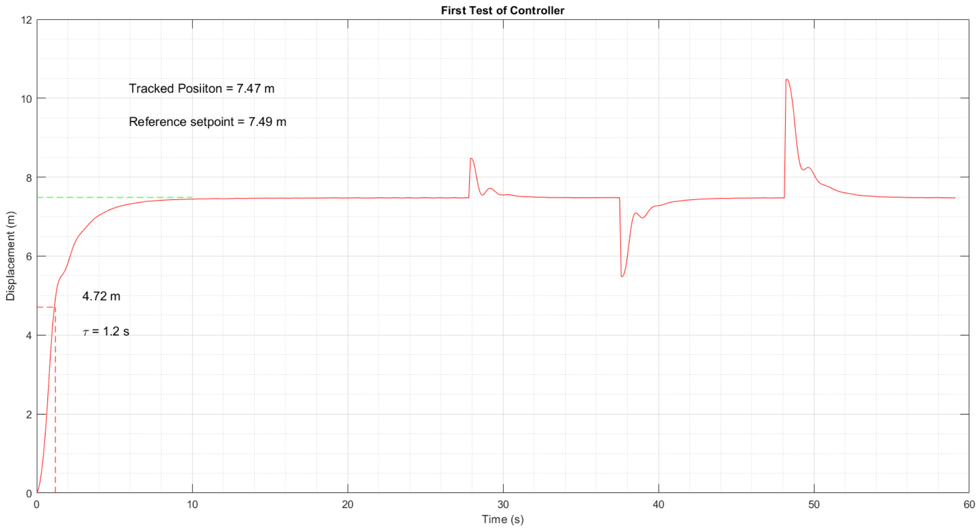


Figure 11: First Controller test

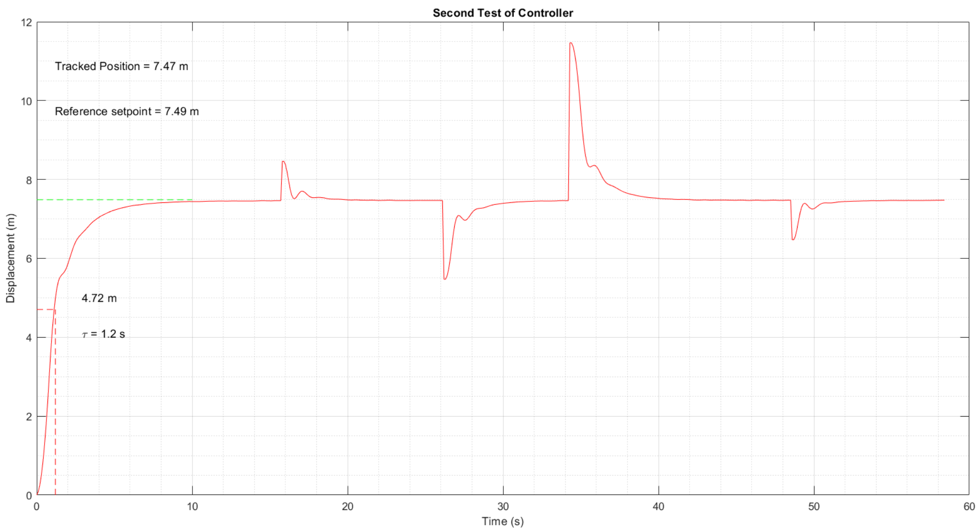


Figure 12: Second Controller test

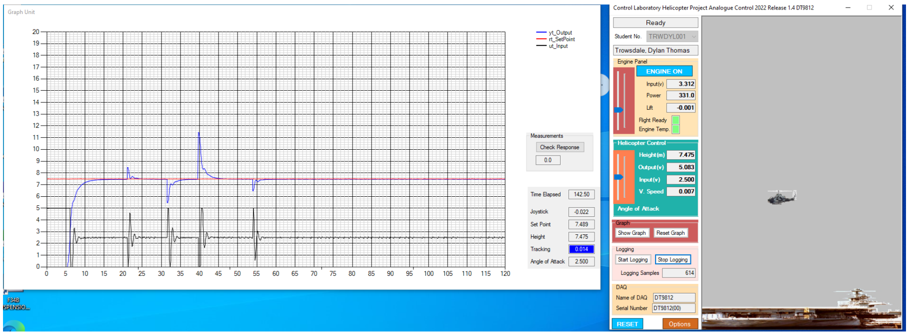

Figure 13: Screenshot of the second test showing the setpoint, the helicopter tracking the setpoint as well as the output disturbances.

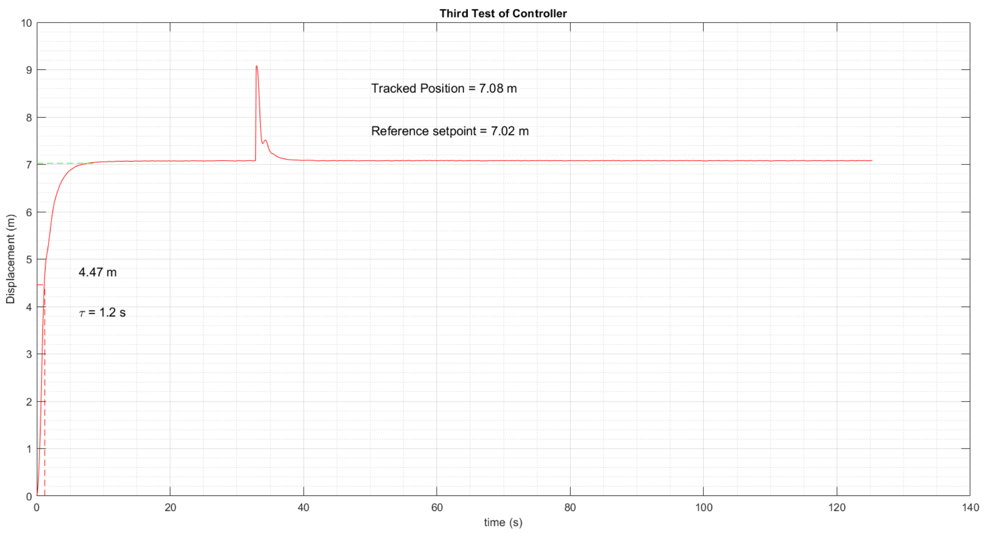


Figure 14: Third Controller test

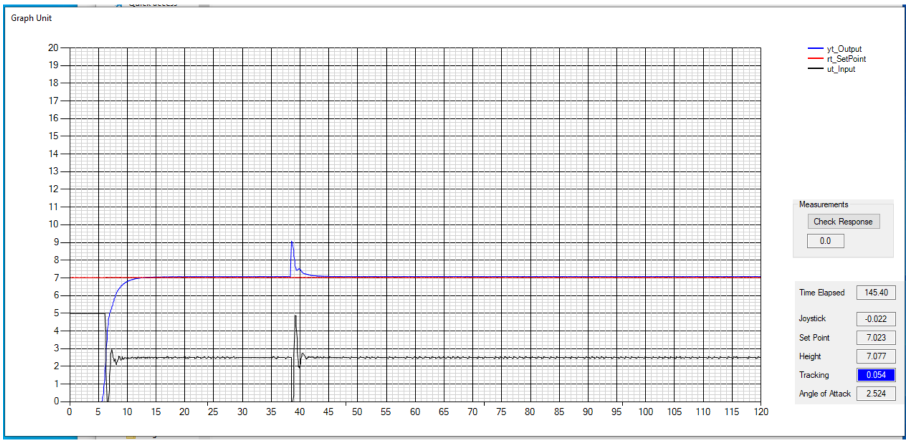


Figure 15: Screenshot of the third test showing the setpoint, the helicopter tracking the setpoint as well as the output disturbances.

## 5.1 Performance Evaluation

It is clear that from all three trials, the controller performed well and within the specifications. There was very little uncertainty between the values calculated between the different trials on the controller as we can see from the data shown in the figures above.

From the three controller tests performed in the controls lab, it is evident that the system was able to meet all the specifications set out for this lab. The settling time for all three trials was $\tau = 1.2s$ which is much faster than the fastest possible response time of the uncompensated system ( $\tau = 11.8s$ ). This easily meets the 20% faster specification which would be: $\tau = 0.8 \times 11.8 = 9.44s$ .

The maximum tracking error of the system was: $\frac{7.08-7.06}{7.06} \times 100 = 0.283\%$. This was well within the Accuracy spec of 90%.

The system did not visibly overshoot the tracking setpoint at all. Therefore, it also meets the 5% overshoot specification.

The system also displayed an ability to handle the effects of output disturbances applied to it. In the real world, this could be the effect of wind, etc. The system was able to recover from these various output disturbances and return to the desired tracking setpoint. in all three trials.

As very precise and accurate resistor values for the controller circuit were chosen, the effects of component tolerances will have a negligible effect on the controller's performance and thus the system meets the specification of robustness to 10% of component values.

Since the controller met all the required design specifications in all the tests, the controller also meets the Robustness to the uncertainty of up to 10% in the aerodynamic constant of the system since $\tau = \frac{1}{b}$. Since the settling time achieved was much faster than the desired settling time specification, even if the aerodynamic constant were to change by 10% the system would easily still meet all the specifications.

# 6 Discussion and Conclusion

From the evaluation of the controller tests, it is evident that the lead-compensator controller designed for the Kitticopter system works as expected and is able to meet all of the design specifications set out in this lab. The controller allowed the Kitticopter to track a reference point set by the user to an accuracy above 99% with no overshoot as well as being able to meet the settling time specifications and robustness specifications. To ensure that the robustness specifications were met, components were carefully selected. Resistor values were carefully measured using the multimeters in the white lab and the components chosen had values well under 1% different from each other. Since the controller's zero and pole were set very far to the left of the plant's poles, it has been proven to be robust to uncertainties in the aerodynamic constant of the system and component tolerances. The controller tests show that the controller still met tracking and overshoot specifications even when the settling times were much faster and even when various output disturbances were applied to the system. This once again proved that the controller was very robust in its design.

In conclusion, this report has dealt with the lead-compensator design of a virtual Kitticopter system that was aimed to allow the Kitticopter to track a user-defined set-point within certain specifications. The goal of the design of the lead-compensator controller was to be able to exceed the specifications that were met by the proportional controller in the previous Kitticopter lab.

The first step involved designing a lead-compensator controller for the virtual Kitticopter system to meet a certain set of specifications. Then, using the Root locus of the open loop transfer function and the Simulation capabilities of Simulink, the controller circuit was simulated and verified for the Kitticopter. Furthermore, the controller circuit was then carefully constructed on a breadboard and validated in the lab. Finally, the circuit was constructed and soldered on veroboard and finally tested again in the lab.

The discussion of the lab results showed that the lead-compensator controller designed and constructed was able to meet all of the specifications required and improved upon the specifications achieved with the proportional controller in the previous lab.

## 6.1 Recommendations for Future Iterations

The controller design was able to meet all the specifications required, however, there were some difficulties encountered in this lab to achieve these results. Potentiometers offer an easy way to adjust voltage levels. Pots were used to control the set point and the input offset voltage. It was often very difficult to control the pot very finely to attain small exact voltages for the input offset voltage. A recommendation would be to use a more reliable method to supply the input offset for example a voltage regulator. The construction of a lead-compensator controller circuit is very difficult in practice and is much more complex than designing it in simulation. Many revisions and trial and error were taken in order to achieve the specifications. A recommendation would be to use a PID controller or to use a digital controller instead of using analog components as it is very hard to achieve the controller transfer functions in practice.

# 7 Appendix and Notes

This lab has given a lot of insight and knowledge into control engineering and has shown the difficulties of building physically implementable controllers in practice.

Bibliography